# Making Mathematics

MATH189AH: Special Topics in Mathematics

Spring 2024

## Contents



## 1 Course Information

- Spring 2024, January 16th–April 25th.
- In-person Tuesdays and Thursdays from 4:15–5:30 PM
- Shanahan Center 2440

### Course description

In this course, students will construct physical and digital objects to illustrate various mathematical concepts. Students will explore topics such as geometry in 2D, 3D, and 4D Euclidean space, non-Euclidean geometry, Fourier analysis, group theory, Galois Theory, combinatorics, probability, and dynamical systems. Emphasizing learning by doing, students will create projects using a variety of technologies and media, such as 3D printers, crochet, pen plotters, compass and straightedge constructions, origami, woodworking, laser cutters, virtual reality software, and more. Students will present their mathematical projects and findings to both the class and the wider public.

### Key principles

- Mathematical exploration, play, curiosity, risk taking, the unknown, and productive failure.
- Communicating mathematical ideas in written, spoken, and artistic form.
- Building and discovering in community.
- Documenting what you learn, what you make, and the things that go wrong.

## Instructor information

Prof. Peter Kagey (KAY-ghee, he/they)
Email: pkagey@g.hmc.edu
Personal phone: (503) 810-6537 (for emergency use)
Grutor office hours: TBA
Student hours: TBA, based on survey responses.
Appointments: `https://calendly.com/profkagey`

I encourage you to come to student hours, grutor office hours, and to make appointments. Let's make things together!

*It is my goal to always respond to emails within one work day. If you haven't heard back from me within 24 hours, please feel free to follow up!*

## Projects & Check-ins

This course will consist of two small projects, one medium project, and two big projects, with weekly check-ins due via Gradescope every Friday evening, and weekly reflections due via Google Forms every Monday evening. According to HMC's catalog, you should be aiming to spend an average of 6.5 hours per week on this course outside of class.

You have up to five[1] extensions that you can use this term, just email me to request one with a proposed due date. If you're finding that you need extensions regularly, we'll work together to make sure you have the support you need for this course.

### Projects

Each project will loosely consist of four phases: explore, prototype, build, publicize. A project will consist of a "soft deadline" where you present your existing progress to the class, and a "hard deadline" where you turn in what you have[2] and prepare to start working on the next project.

### Check-ins

Each week you will have a check-in which will be compiled as a LaTeX document and submitted via Gradescope. Every check-in will have you fill out a self-reflection via a Google Form. A check-in might also have you install software on a computer, solve a math problem, read and respond to an essay, explore a website and talk about what you find, and/or reflect on your peers' project presentations. (The first three check-ins will look more like traditional "homework" assignments and the remaining eleven check-ins will look more like "progress reports.")

## Grading

This course will use an ungrading scheme, where feedback is provided each week, and you are asked to self-assess your grade each week. During this term, you will receive feedback from the instructor, from peers, from our graders, and from self-reflection.

At the end of the term, I will give you course-level feedback and a list of the grades you have given yourself along the way, after which I will ask you to write a reflection on this course and propose the grade that seems appropriate to you.

[1] Five *or more*
[2] typically in the form of a blog post, video, or another public-facing expression

## Resources and policies

### Access and Accommodations

HMC is committed to providing an inclusive learning environment and support for all students. Students with a disability (including mental health, chronic or temporary medical conditions) who may need accommodations in order to fully participate in this class are encouraged to contact the Office of Accessible Education at access@g.hmc.edu to request accommodations. Students from the other Claremont Colleges should contact their home college's Accessible Education officer.

### Diversity and Inclusion

Harvey Mudd College values diversity and inclusion; we are committed to a climate of mutual respect and full participation. Our goal is to create learning environments that are usable, equitable, inclusive, and welcoming. If there are aspects of the instruction or design of this course that result in barriers to your inclusion or accurate assessment or achievement, please notify the instructor as soon as possible.

### Title IX

If I learn of any potential violation of our gender-based misconduct policy, I am required to notify the HMC Title IX Coordinator, Danny Ledezma. If students want to speak to someone confidentially, the resources listed below are available, and more can be found on the Title IX page of the HMC website. (Speaking with a confidential resource does not preclude students from making a formal report to the Title IX Coordinator at a later time.)

- EmPOWER CENTER
- Monsour Counseling and Psychological Services
- McAlister Center of the Chaplains

### Campus resources

College students often experience issues that may interfere with academic success such as academic stress, sleep problems, juggling responsibilities, life events, relationship concerns, or feelings of anxiety, hopelessness, or depression. If you or a friend is struggling, we strongly encourage you to seek support. Helpful, effective resources are available on campus, at no charge.

- If you are struggling with this class, please visit during office hours or contact me by email.
- Check-in with an academic dean if you are struggling in courses or unsure what academic resources are available at HMC by emailing academicdeans@g.hmc.edu.
- Reach out to the Office of Health and Wellness to discuss options available to by emailing wellness@g.hmc.edu.
- MCAPS provides crisis support services 24/7/365. Students can call us at 909-621-8202 and press "1" at the prompt to speak with a crisis counselor. Other prompt options are available for those not in crisis.

### Writing Center

The HMC Writing Center provides a welcoming space for writers to get feedback on their composition projects, whether written, spoken, or visual pieces. The Writing Center is located both online and in Shanahan 1470.

## Academic Honesty and the Honor Code

Collaboration with other students on individual assignments is encouraged and in some cases required. Students must each turn in their own assignment, but collaborating on the written portion of an assignment with other students is acceptable, as long as you include proper attribution.

You are encouraged to use any tools and resources that you have access to, including your professors, your classmates, Google, ChatGPT, Github, etc. It is always important to give attribution for ideas that are not your own.

All students enrolled in this course are bound by the HMC Honor Code. More information on the HMC Honor Code can be found in the HMC Student Handbook.

# 2 Calendar

## Week 1: Making *Making Mathematics*

*January 16th and January 18th* (HW 0 due on Friday @ 6PM)

**Tuesday.** Make!

- Explore campus, make and photograph mathematics, discover math in the world.

**Thursday.** Class policies and goals discussion.

- What goals should we have for the class?
- What kinds of constraints will help you to be successful?
- What do you want weekly check-ins to look like? Project deliverables?
- What math topics do you want to learn about from me? (e.g. Knot theory, etc.)
- What do you want classroom time to look like?
- How can use use our Discord channel most effectively?

## Week 2: Making Curves

*January 23rd and January 25th* (HW 1 due on Friday @ 6PM)

**Tuesday.** Lecture/workshop: Geometry in 2D and 3D.

- Vectors and angles.
- Planes, normal vectors, and cross products.
- Parametric curves.
- Bézier curves.
- Tangent vectors, re-parameterization, curvature, etc.

**Thursday.** Read and wonder

- *Math Horizons*
- *The American Mathematical Monthly*
- *Mathematics Magazine*
- *The College Mathematics Journal*
- *The Mathematical Intelligencer*
- *Mathematical Gazette*

## Week 3: Making New Ideas

*January 30th and February 1st* (HW 2 due on Friday @ 6PM)

**Tuesday.** Tour of the Hive!

**Thursday.** Discuss Project 0.A.

- PIL
- SVGs
- TikZ
- Cricut/AxiDraw V3/Laser Cutter

## Week 4: Making Surfaces

*February 6th and February 8th* (Project 0.A due on Friday @ 6PM)

**Tuesday.** Polyhedra, surfaces, and solids

- OpenSCAD
- Polyhedra, duals, Conway notation
- Solids of rotation
- Infinite area, finite volume

**Thursday.** Tour of the Makerspace!!

## Week 5: Making Symmetry

*February 13th and February 15th* (Project 0.B due on Friday @ 6PM)

**Tuesday.** Discussion of how Project 0.A went.

- Wallpaper groups
- Groups of polyhedra
- Coxeter groups (Mirrors and kaleidoscopes)
- Point groups in 2D/3D
- Braid groups `https://youtu.be/3vZ8TT5Ln7o`

**Thursday.** Visit *Seeing the Unseen: Math and Art* at the Wignall Museum of Contemporary Art.

## Week 6: Making progress

*February 20th and February 22nd* (Project 1.A check-in due on Friday @ 6PM)

**Tuesday.** Show and tell for Projects 0.A & 0.B.
Share Project 1 plans

**Thursday.** Guest class on Mathematica by Prof Yong

## Week 7: Making stories

*February 27th and February 29th (!!)* (Project 1.B check-in due on Friday @ 6PM)

**Tuesday.** Library tour

**Thursday.** Project 1 presentations

## Week 8: Making a giant polyhedral structure?!

*March 5th and March 7th* (Project 1.C due on Friday @ 6PM)

**Tuesday.** Spherical geometry.

- Geodesics, lengths, and angles
- Polyhedra & Polytopes
- $V - E + F$

**Thursday.** Collaborative build with Studio Infinity

## Spring break

*March 12th and March 14th*

## Week 9: Making tapestries

*March 19th and March 21st* (Project 2.A check-in due on Friday @ 6PM)

**Tuesday.** Mid-term check-in

- Regular and uniform tilings
- Penrose tilings
- Aperiodic monotile

**Thursday.** Hyperbolic geometry

- Poincaré disk model.
- Hyperbolic trig functions.
- Tiling the plane.
- *Make Schlafli symbol* $\{3, 7\}$ *with various media.*

## Week 10: Making our own luck

*March 26th and March 28th* (Project 2.B check-in due on Friday @ 6PM)

**Tuesday.** Probability

- Buffon's needle problem
- Galton board
- Random walk
- Fair dice

**Thursday.** Project 2 presentations

## Week 11: Making connections

*April 2nd and April 4th* (Project 2.C due on Friday @ 6PM)

**Tuesday.** Graph theory

- Mazes and spanning trees
- Planar graphs
- Eulerian walks
- 1-skeletons of polyhedra

**Thursday.** Polyominoes and their generalizations

- Polyforms and packing problems
- Directed animals

## Week 12: Making a list (and checking it twice)

*April 9th and April 11th* (Project 3.A check-in due on Friday @ 6PM)

**Tuesday.** (Combinatorics part 1)

**Thursday.** (Combinatorics part 2)

## Week 13: Making change

*April 16th and April 18th* (Project 3.B due on Friday @ 6PM)

**Tuesday.** Differential equations

- Systems of ODEs
- Chaos!

**Thursday.** Project 3 presentations

## Week 14: Making it all come together

*April 23rd and April 25th* (Final portfolio due on Friday @ 6PM)

**Tuesday.** TBD

**Thursday.** End of term celebration!

# A Technologies & Media

**Fiber arts**

- Digital Jacquard loom
- Manual loom
- Sewing machine
- Leather sewing machine
- Embroidery machine
- Crochet
- Knitting
- Felting

**Makerspace Laser3D room**

- Laser cutter
- Waterjet cutter
- 3D printer
  - FDM
  - SLA (resin)
  - SLS (nylon)

**Software**

- PIL (Python)
- OpenSCAD
- Inkscape
- Blender
- Unity
- Mathematica
- Rhino3D

**Wood shop**

- Wood lathe
- Drill press
- Table saw
- Miter saw
- CNC router
- Sanders

**Machining**

- Metal Lathe
- Mill
- Shear
- Bandsaw
- Welding
- Bending brake

**Digital**

- Video camera
- Camera
- Projector
- Stop-motion
- Animated GIFs
- VR headsets
- Microphones; speakers
- Large format printer

**Electrical**

- Electric circuits
- Oscilloscope
- Soldering

**Paper and printing**

- Origami
- Compass; straightedge
- Papier-mâché
- Pop-up cards
- Printing press
- Linocut and woodcut printing
- Screen printing

**Other technologies**

- LEGO, K'nex, Zometool
- Bead art
- Lite Brite
- Jewlery making
- Modeling clay
- Ceramics
- Balloon animals
- Cricut vinyl cutter
- Scrapbooking
- Pen plotters

# B Project ideas

1. Make a logical circuit out of something interesting to add two three bit numbers.[3]
2. Interesting drawing with compass and straightedge, and perhaps multiple colors of pens.
3. Papier-mâché a genus $k$ surface.
4. Illustrate all of the wallpaper groups by making actual examples.
5. Make kaleidoscopes that illustrate Coxeter groups. [4]
6. Put mirrors inside of polyhedra.[5]
7. Cut and fold paper and cut a corner to cut out a shape of your choosing.[6]
8. Make a pop-up book.[7]
9. Hinged dissections.[8]
10. Dissections of polyhedra.[9]
11. Burr Puzzles with identical pieces.[10]
12. Prince Rupert's cube.
13. Fourier series to draw pictures. Can you make a mechanical version that makes a nice picture? [11]
14. A physical implementation of Moser's worm problem[12] or of the moving sofa problem.[13].
15. A physical implementation of a Kakeya set.
16. Make your own version of Spot-It![14]
17. Make playing cards, where each is illustrated with an example of something from a set of size 52 or from four sets of size 13.[15]
18. Draw a fractal in PIL.
19. 3D print versions of Blokus for other geometries, like all of the polyforms on a Cairo pentagonal tiling.[16]
20. Illustrate ideas in algebraic topology by making a torus with a nail in it and (stretchy) string looping to and from the nail.[17]
21. Make circles and spheres based on other metrics.[18]
22. Make generative art that uses ideas from gradient descent. (e.g. a spline where mean curvature has certain properties.)
23. Make "vertices" for polyhedra so that you can make big shapes with dowels.[19]
24. All knots with a given number of crossings.
25. Implement the brachistochrone curve vs drawings that you have random people draw that they think would be fastest.[20]
26. Explore the tautochrone curve or a simple pendulum by showing that time is approximately independent of distance.[21]
27. Laser cut wood so that we can make interesting 3D objects, working with the constraint that all of the pieces meet at right angles.
28. Make a pseudosphere on a lathe or using another technology. [22]

[3] `https://www.youtube.com/watch?v=lNuPy-r1GuQ`
[4] Geometiles Kaleidoscopes
[5] *Portals* by Anthony James and Archimedean Billiards with Geometiles
[6] Katie Steckle's favorite theorem
[7] *Pop-Up Geometry* by Joseph O'Rourke and *4 Views of the 4-Cube* by Richard Hammack
[8] Laura Taalman, 3D printed Hinged Dissections and Foldable Polyhedra
[9] Four-piece Tetrahedron, Two-piece Tetrahedron
[10] "Symmetric Stick Puzzles" by George W. Hart, Star Puzzle, 12-piece burr puzzle with identical pieces, Segerblom knot
[11] "Epicycles, complex Fourier series and Homer Simpson's orbit" by Mathologer
[12] "Moser's worm problem", Wikipedia
[13] "Moving sofa problem", Wikipedia
[14] "The Mind-Bending Math Behind Spot It!, the Beloved Family Card Game", Smithsonian Magazine
[15] `https://oeis.org/search?q=52`
[16] `https://boardgamegeek.com/boardgame/21550/blokus-trigon`
[17] `https://en.wikipedia.org/wiki/Fundamental_group`
[18] *The 4-Norm's Hidden 2-Norm*
[19] `https://www.thingiverse.com/thing:2805280`
[20] `https://en.wikipedia.org/wiki/Brachistochrone_curve`
[21] `https://en.wikipedia.org/wiki/Tautochrone_curve`
[22] `https://en.wikipedia.org/wiki/Pseudosphere`

29. Make solids of revolution, compute their volume with calculus, then compute the volume by placing it underwater.

30. Build a Galton board where the pieces are not all the same shape or introduce other kinds of bias. [23]

31. Build slices of a 4 dimensional object in the woodshop (analogous to how slices of a tetrahedron can be all triangles or all rectangles, or something in between.)

32. Build the 1-skeleton of the projection of 4-polytopes like the 24-cell.

33. Create a VR setup where we can see the 3D shadow of 4D objects as it moves in certain ways.

34. Make a ruled surface using string and peg boards.

35. Design a pattern for the Jacquard loom that includes every $n \times n$ square where no $k$ vertically/horizontally adjacent cells are the same color.

36. Draw a space-filling curve with the AxiDraw V3.

37. Illustrate a video of flying through the stars in perspective, so that some small circles are far away and move slowly, and some are close and move/grow quickly.

38. Design a puzzle such as "Crystal Ball" by Pavel Curtis[24], or

39. Make a Towers of Hanoi variant and analyze it.

40. Make a variant of the "Pentomino" puzzle by Solomon Golomb with other polyforms.

41. Penrose tilings and aperiodic tilings and the Einstein tile.

42. Make a hexaflexagon.

43. Five intersecting tetrahedra.[25]

44. Make some Heesch Tilings.[26]

45. Seven touching cylinders.[27]

46. Make interesting gears and mechanical linkages.[28]

47. Snake cube puzzle generalizations.

48. Make generalizations of the Boerdijk-Coxeter helix.[29]

49. Illustrate the four-color theorem in different topological settings.[30]

50. Make polyhedra and dual polyhedra.

51. Schönhardt polyhedron and triangularization.[31]

52. Eulerian circuits with a pen plotter.

53. Truchet tile jigsaw puzzles.

54. Make a Gömböc.[32]

55. Realize a maze as a spanning tree of a graph.

56. Chladni plates [33]

57. Make a Tchotchke shelf based on "Squaring the Square" and Kirchhoff's circuit laws.[34]

58. Make a Gyroid.[35]

59. Illustrate a cellular automaton like Rule 30.[36]

60. Brick stacking problem. Enumerate all possible stable brick stacks with $n$ bricks.

61. Make a poster enumerating all Catalan objects of size 6 on large format printer.

62. Build all of the resistances you can make with six $1\Omega$ resistors.

63. Build a double pendulum to study chaos

[23] https://www.youtube.com/watch?v=zeJD6dqJ5lo
[24] http://www.pavelspuzzles.com/2009/08/star_czech.html
[25] https://www.youtube.com/watch?v=KB72GPGET80
[26] "Heesch's Tiling Problem" by Casey Mann
[27] https://www.mathpuzzle.com/index2013.html
[28] e.g. the trammel of Archimedes.
[29] https://en.wikipedia.org/wiki/Boerdijk%E2%80%93Coxeter_helix
[30] Szilassi polyhedron and https://en.wikipedia.org/wiki/Four_color_theorem#Generalizations
[31] https://en.wikipedia.org/wiki/Sch%C3%B6nhardt_polyhedron
[32] https://en.wikipedia.org/wiki/G%C3%B6mb%C3%B6c
[33] https://youtu.be/rjueHI002Fg
[34] https://en.wikipedia.org/wiki/Squaring_the_square
[35] https://en.wikipedia.org/wiki/Gyroid
[36] https://en.wikipedia.org/wiki/Rule_30

64. Build a variation on a Spirograph.[37]

65. Build a harmonograph or Blackburn pendulum to make Lissajous curves and other illustrations[38]

66. Construct a tensegrity structure.[39]

67. Illustrate Pappus's centroid theorem, Morley's trisector theorem, or another geometric idea.

68. Make a cookie cutter of a shape that tiles the plane, like the "spectre" tile (an aperiodic monotile).

[37] https://en.wikipedia.org/wiki/Spirograph
[38] https://en.wikipedia.org/wiki/Harmonograph
[39] https://en.wikipedia.org/wiki/Tensegrity

# C Links and Inspiration

**Bathsheba Grossman**

- Wikipedia
- Shapeways
- Website

**Daina Taimiņa**

- Talk at TEDxRiga
- *Crocheting Adventures with Hyperbolic Planes*

**Laura Taalman**

- Hacktastic blog

**Henry Segerman**

- Shapeways
- YouTube

**Dave Bachman** (Pitzer math professor)

- Math/Art Blog
- Shapeways

**SOME (Summer of Math Exposition)**

- SOME1 results
- SOME2 results
- SOME 3 video entries
- SOME 3 non-video entries

**Wikipedia**

- Mathematics and art
- Mathematics and fiber arts
- Mathematics and architecture
- Mechanical puzzle

**Others**

- Lea Albaugh, "'It's Just Matrix Multiplication': Notation for Weaving"
- BonitumART on Etsy wood cut surfaces
- Bridges Art Galleries
- Gathering 4 Gardner YouTube channel
- GrapefruitGecko Youtube channel.
- George Hart's "Geometric sculpture"
- Kosticks welded polyhedra
- mathpuzzle blog by Ed Pegg Jr.
- George Sicherman's "Polyform Curiosities"
- *Spherical Models* by Magnus J. Wenninger